

6666 Core Mathematics C4 – June 2005

1. $2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3 + \dots$
2. $(2, -2), (-2, 2)$
3. (a) $\frac{3}{2x-3} + \frac{1}{x+2}$ (b) $\ln 54$
4. $\frac{1}{\sqrt{3}}$
5. (a) $\frac{1}{4} + \frac{1}{4}e^2$ (b) $0.89022; 3.96243$ (c) 2.168
6. (a) $\frac{-2\sin t \cos t}{\operatorname{cosec}^2 t}$ (b) $y - 1 = -\frac{1}{2}(x - 2)$
(c) $y = \frac{8}{4+x^2}; x \dots 0$
7. (a) $(2, 2, -2)$ (b) $\frac{1}{3}$ (d) $6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$
8. (c) $\frac{75}{\ln 2}$

6666 Core Mathematics C4 – January 2006

1. $5y + 4x + 6 = 0$
2. (a) $1.01959; 1.08239; 1.41421$ (b) 0.8859 (c) 0.51%
3. 16
4. $\pi \left[\frac{13}{4}e^6 - \frac{1}{4}e^2 \right]$
5. (a) $A = 3; C = 4$ (b) $4 + 8x + 27\frac{3}{4}x^2 + 80\frac{1}{2}x^3 + \dots$
6. (a) $a = 18; b = 9$ (b) $(6, 10, 16)$ (c) $14\sqrt{2}$
7. (a) $4\pi r^2$ (b) $\frac{1000}{4\pi r^2(2t+1)^2}$ (c) $500(1 - \frac{1}{2t+1})$
(d) (i) 4.77
8. (c) $4\pi + 3\sqrt{3}$

6666 Core Mathematics C4 – June 2006

1. $7x + 2y - 2 = 0$
2. (a) $A = -\frac{3}{2}; B = \frac{1}{2}$ (b) $-1 - x + 0x^2 + 4x^3$
3. (a) 12 (b) 88.8264
4. (a) $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - \frac{1}{2})$
5. (a) $a = -5$ and $b = 11$ (b) $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ (c) $2 : 3$
6. (a) $0.5 \ln 1.5; 1.5 \ln 2.5$ (b) (i) 1.792 (ii) 1.684
7. (c) 6; $t = 3$
8. (c) $\frac{75}{\ln 2}$

6666 Core Mathematics C4 – January 2007

1. (a) $\frac{1}{4} + 1\frac{1}{4}x + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$
2. (a) $\frac{\pi}{12}$ (b) $\frac{16\pi}{3} \text{ cm}^3$
3. (a) $\frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$ (b) $y = \frac{1}{\sqrt{3}}x$
4. (a) $\frac{-1}{(x-1)} + \frac{4}{(2x-3)}$
(b) $\ln y = -\ln(x-1) + 2 \ln(2x-3) + c$
(c) $y = \frac{10(2x-3)^2}{(x-1)}$
5. (a) $\frac{dy}{dx} = \frac{\cos x}{\sin y}$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$
6. (b) $64 \ln 2$
7. (a) $3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ (c) $\frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ (d) 109.5°
8. (a) $e^{\sqrt{7}}, e^{\sqrt{10}}, e^{\sqrt{13}}$ (b) 110.6 (d) $2e^4$

6666 Core Mathematics C4 – June 2007

1. $\frac{1}{27} - \frac{2x}{27} + \frac{8x^2}{81} - \frac{80x^3}{729}$
2. $\frac{1}{6\ln 2}$
3. (a) $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$
 (b) $\frac{1}{4}x \sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 (+c)$
4. (a) $A = 2, B = -2, C = 2$ (b) $2 + \ln\left(\frac{9}{5}\right)$
5. (b) $\cos \theta = \frac{7}{10}$
6. (a) $\frac{\cos^4 t}{2 \sin t}$ (b) $y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$ (c) $y^2 = \frac{x}{1+x}$
7. (a) 0.44510, 0.64359, 0.81742 (b) 0.4726 (c) $\pi \ln \sqrt{2}$
8. (a) $P = P_0 e^{kt}$ (b) $t = 399$ minutes (c) $P = P_0 e^{\sin \lambda t}$
 (d) 441 minutes
- 6666 Core Mathematics C4 – January 2008**
1. (a) 1.84432, 4.81048, 4.81047 (b) 12.1948
2. (a) $2 - \frac{1}{4}x - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots$ (b) 1.9746810
3. $\frac{\pi(b-a)}{(2a+1)(2b+1)}$
4. (i) $x \ln \frac{x}{2} - x + c$ (ii) $\frac{\pi}{8} + \frac{1}{4}$
5. (a) $y = 16$, $y = 8$ (b) $-3, 0$
6. (a) $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ (b) $\mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ (c) $\frac{\pi}{4}$
7. (b) $\ln \frac{3}{2}$ (b) $y = \frac{1}{e^x - 1}$ (c) $x > 0$
8. (d) $2000 \ln 2 - 1000$ (b) 6 minutes 26 seconds

6666 Core Mathematics C4 – June 2008

1. (a) $e^{0.32}$, $e^{1.28}$ (b) 4.922
2. (a) $xe^x - e^x (+c)$ (b) $x^2 e^x - 2(x^2 e^x - e^x) (+c)$
3. (a) $0.00254 \text{ cm s}^{-1}$ (b) $0.48 \text{ cm}^3 \text{ s}^{-1}$
4. (a) $y - 2x = 0$ (b) (2, 4) and (-2, -4)
5. (a) $\frac{1}{2}[1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots]$ (b) $4 + 2x + \frac{33}{32}x^2$
6. (d) $-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}$
7. (a) $\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$ (b) $\sec^2 x = \frac{8+4y}{2-y}$
8. (a) $t = \frac{\pi}{3}, 0 \leq t \leq \frac{\pi}{2}$ (d) $\frac{64}{3} - 8\sqrt{3}$

6666 Core Mathematics C4 – January 2009

1. (a) $\frac{3x^2}{2y-3}$ (b) 4
2. (a) 3 (b) $\frac{9}{4}\pi \ln 9$
3. (a) $B = 4$ (b) $4 + 0x + \frac{39}{4}x^2$ (c) 1.1%
4. (b) $p = 1$ (c) $\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ (d) $\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$
5. (b) $\frac{1}{8\pi}$
6. (a) $\tan x - x + c$ (b) $-\frac{1}{2x^2} \ln x + \frac{1}{2} \left(-\frac{1}{2x^2} \right) + c$
7. (a) (7, 1) (c) $\left(\frac{441}{8}, \frac{81}{4} \right)$

6666 Core Mathematics C4 – June 2009

1. (a) $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$
2. (a) 1.14805 (b) 8.884 (c) 9
3. (a) $A = 4, B = -3, C = 1$
 (b) (i) $\frac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + C$ (ii) $\ln\left(\frac{125}{81}\right)$
4. (a) $\frac{dy}{dx} = \frac{2+2ye^{-2x}}{e^{-2x}-2y}$ (b) $x-4y+4=0$
5. (a) $-\frac{\sqrt{3}}{2}$ (b) $y = \sqrt{(18-9x)} \quad (= 3\sqrt{(2-x)}), k = 2$
 (c) $0 \leq f(x) \leq 6$
6. (a) $-\frac{2}{3}(5-x)^{\frac{3}{2}} + c$ (b) (i) $-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} (+C)$
 (b) (i) $-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} (+C)$ (ii) $\frac{128}{15} \left(= 8\frac{8}{15} \approx 8.53 \right)$
7. (a) $\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ (b) $\sqrt{126}$ or 11.2
 (c) 36.7° (d) $d = 3\sqrt{5} (\approx 6.7)$ (e) 30.1 or 30.2
8. (a) $x = \tan \theta \Rightarrow \frac{dx}{d\theta} = \sec^2 \theta$
 (c) $\frac{4}{3}\pi^2 - 2\pi\sqrt{3}$ where $p = \frac{4}{3}, q = -2$

6666 Core Mathematics C4 – January 2010

1. (a) $1 - 4x - 8x^2 - 32x^3 - \dots$ (c) 4.79584
2. (a) 1.386, 2.291 (b) 7.37
 (c) (i) $\frac{x^2}{2} \ln x - \frac{x^2}{4} (+c)$ (ii) $\frac{1}{4}(64 \ln 2 - 15)$
3. (a) $\frac{2 \sin 2x}{3 \sin 3y}$ (b) $\frac{\pi}{9}$ (c) $6x + 9y - 2\pi = 0$
4. (a) $(-6, 4, -1)$ (b) $\frac{19}{26}$ (c) $(10, 0, 11)$
 (d) $\begin{pmatrix} 16 \\ -4 \\ 12 \end{pmatrix}$ (f) 27.9
5. (a) $9x + 6 \ln x (+c)$ (b) $y^2 = (6x + 4 \ln x - 2)^2$
6. 0.299 cm s^{-1}
7. (a) 4, 41 (b) 648
8. (a) $\frac{\sqrt{3}-1}{4}$ (b) $16\pi \left(\frac{\sqrt{3}-1}{4} \right)$
- 6666 Core Mathematics C4 – June 2010**
1. (a) 1.2247, 1.1180 (b) (i) 1.249 (ii) 1.257
3. $\frac{dy}{dx} = 4 \ln 2 - 2$
4. (a) $\frac{dy}{dx} = \frac{\sec^2 t}{\sin t \cos t}$ (b) $x = \frac{3}{8}$
5. (a) $A = 2, B = -1, C = 4$ (b) $5 + \frac{3}{2}x^2 + \dots$
6. (b) $\frac{\pi^2}{16} - \frac{7}{4}$
7. (a) $(5, 9, -1)$ (b) 57.95° (c) $15\sqrt{5}$
8. (b) $t = 15 \ln 2$

6666 Core Mathematics C4 – January 2011

1. $\frac{\pi}{4}$
2. $\ln 4$
3. (a) $\frac{1}{x-1} - \frac{3}{3x+2}$ (b) $\ln(x-1) - \ln(3x+2) + c$
(c) $y = \frac{64(x-1)}{3x+2}$
4. (a) $-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ (b) $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$
(c) -6 (d) $\sqrt{46}$
5. (a) $\frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3$ (b) $a = -1, b = 3$
(c) $\frac{27}{16}$
6. (a) $y - 7 = -\frac{1}{18}(x - \ln 3)$ (b) $y = e^{2x} - 2$
(c) $\pi(36 + 4 \ln 2)$

7. (a) 0.1847, 0.1667 (b) 0.543 (c) $2 + 8 \ln \frac{5}{6}$

6666 Core Mathematics C4 – June 2011

1. $A = 4, B = 3, C = 1$
2. $f(x) = \frac{1}{3} - \frac{2}{9}x^2 + \frac{2}{81}x^4$
3. (a) 0.04π (b) $\frac{1}{32}$
4. (a) 0.0333, 1.3596 (b) 1.30 (d) $R = \frac{1}{2}(2 \ln 2 + 1)$
5. $16(2 + 2 \ln 2)$

6. (a) $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ (b) 69.1° (d) 6.99
7. (a) $\theta = \frac{\pi}{3}$ (b) $k = \frac{17}{16}$ (c) $\sqrt{3}\pi - \frac{1}{3}\pi^2$
8. (a) $\frac{1}{2}(4y+3)^{\frac{1}{2}} + c$ (b) $y = \frac{1}{4} \left(2 - \frac{2}{x} \right)^2 - \frac{3}{4}$

6666 Core Mathematics C4 – January 2012

1. (a) $-\frac{4}{9}$ (b) $9x - 4y + 13 = 0$
2. (a) $-\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + c$
(b) $-\frac{1}{3}x^2 \sin 3x + \frac{2}{9}x \cos 3x + \frac{2}{27} \sin 3x + c$
3. (a) $\frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2$ (b) $k = -3$ (c) $A = 5 \frac{5}{8}$
4. $\frac{\pi}{3} \ln 4$
5. (a) $\frac{-6 \sin 2t}{4 \cos \left(t + \frac{\pi}{6} \right)}$ (b) $(2, 3), (2\sqrt{3}, -3), (-2, 3), (-2\sqrt{3}, -3)$
6. (a) 0.73508 (b) 1.1504 (d) 0.077
7. (a) $3\mathbf{i} + 3 + 5\mathbf{k}$ (b) $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ (d) $2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$
(e) 23.2 (f) 3.54
8. (a) $\frac{1}{5P} + \frac{1}{5(5-P)}$ (b) $P = \frac{5}{(1+4e^{-\frac{1}{3}t})}$ (c) $1 + 4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$

6666 Core Mathematics C4 – June 2012

1. (a) $A = 1$, $B = -3$, $C = 3$
(b) (i) $\ln x - \ln(3x-1) - \frac{1}{3x-1} + c$ (ii) $\frac{3}{10} + \ln \frac{4}{5}$
2. (b) $0.000\ 25 \text{ cm s}^{-1}$ (c) $0.024 \text{ cm}^2 \text{ s}^{-1}$
3. (a) $2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3$
(b) $g(x) = 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3$
(c) $h(x) = 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3$
4. $\frac{1}{2}y^2 = 3 \tan x - 1$
5. (a) $\frac{18-6xy}{16y^2+3x^2}$ (b) $(2, \frac{3}{2}), (-2, -\frac{3}{2})$
6. (b) $y = 2x - 2$ (c) $x^2 = 12\left(1 - \frac{y}{4}\right)\frac{y}{4}$
7. (a) 7.49 (b) $\frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}} + c$ (c) $\frac{46}{3} \ln 2 - \frac{28}{9}$
8. (a) $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$

6666 Core Mathematics C4 – January 2013

1. $+\frac{27}{16}x^2 - \frac{135}{32}x^3 + \dots$
2. (a) $-\frac{1}{2x^2} \ln x + \frac{1}{2} \left(-\frac{1}{2x^2} \right) + c$
(b) $\frac{3}{16} - \frac{1}{8} \ln 2$
3. $3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$
4. (a) 1.0981 (b) $R = 2.843 \text{ units}^2$
(c) $R = \frac{11}{3} + 2 \ln 2 - 2 \ln 3 \text{ units}^2$
5. (b) $x = 1$ (c) $y - 3 = \frac{1}{8 \ln 2}(x - 0)$
(d) $R = \frac{15}{2 \ln 2} - 2 \text{ units}^2$
6. (a) $x = \frac{\pi}{3}, \frac{5\pi}{3}$ (b) $4\pi^2 + 3\pi\sqrt{3}$
7. (a) $\mathbf{r} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$ (b) 69.1° (c) $\begin{pmatrix} 9\frac{1}{3} \\ 14\frac{1}{3} \\ -3\frac{2}{3} \end{pmatrix}$
8. (b) 77 minutes

6666 Core Mathematics C4 – June 2013 (R)

1. $\frac{2}{(2x+1)} - \frac{1}{(x+1)} + \frac{2}{(x+1)^2}$
2. $\frac{dy}{dx} = \frac{1}{5} \ln(3e^3)$
3. $2 + 2 \ln\left(\frac{3}{5}\right)$
4. (a) $2 - \frac{3}{4}x; -\frac{9}{32}x^2 - \frac{45}{256}x^3 + \dots$ (b) $x = 19.2201$
5. (a) $x = 6.248$ (b) $R = 49.37$
 (c) $R = 60 - 132e^{-\frac{8}{3}}$ (d) 1.46
6. (a) $a = -3$ (b) $b = -1, c = -4$
 (c) $|AB| = 13$ (d) $\overrightarrow{OB'} = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$
7. (a) $\frac{4}{72}$ (b) $27 \leq x \leq 216$ (c) 847.2π
8. (b) $x = M - M e^{-kt}$ (c) $x = \frac{2}{3}M$

6666 Core Mathematics C4 – June 2013

1. (a) $x^2e^x - 2(xe^x - e^x)$ (b) $e - 2$
2. (b) $\frac{7025}{4056}$
3. (a) 1.154701 (b) 1.7787 (c) 2π
4. (a) 1 (b) $y = 2\left(\frac{x}{2}\right)^2, k = 2$ (c) $0 \leq f(x) \leq 2$
5. (b) $2 \ln\left(\frac{5}{3}\right)$
6. (b) 161 seconds
7. (a) $\frac{dx}{dy} = \frac{-2x - 4y}{4x + 2y}$ (b) $x = -3, y = 6$
8. (a) $p = 1$ (b) $\begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -6 \\ 8 \end{pmatrix}$